NEUTRINO OSCILLATIONS WITH FOUR GENERATIONS

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Recent status of neutrino oscillation phenomenology with four neutrinos is reviewed. It is emphasized that the so-called (2+2)-scheme as well as the (3+1)-scheme are still consistent with the recent solar and atmospheric neutrino data.

1 Introduction

There have been several experiments [1,2,3,4,5,6,7,8,9 10,11,12,13] which suggest neutrino oscillations. To explain the solar, atmospheric and LSND data within the framework of neutrino oscillations, it is necessary to have at least four kinds of neutrinos. It has been shown in the two flavor framework that the solar neutrino deficit can be explained by neutrino oscillation with the sets of parameters $(\Delta m^2_\odot, \sin^2 2\theta_\odot) \simeq (\mathcal{O}(10^{-5}\text{eV}^2), \mathcal{O}(10^{-2}))$ (SMA (small mixing angle) MSW solution), $(\mathcal{O}(10^{-5}\text{eV}^2), \mathcal{O}(1))$ (LMA (large mixing angle) MSW solution), $(\mathcal{O}(10^{-7}\text{eV}^2), \mathcal{O}(1))$ (LOW solution) or $(\mathcal{O}(10^{-10}\text{eV}^2), \mathcal{O}(1))$ (VO (vacuum oscillation) solution). At the Neutrino 2000 Conference the Superkamiokande group has updated their data of the solar neutrinos and they reported that the LMA MSW solution gives the best fit to the data [9]. At the same time they also showed that the scenario of pure sterile neutrino oscillations $\nu_e \leftrightarrow \nu_s$ is excluded at 95%CL. It has been known that the atmospheric neutrino anomaly can be accounted for by dominant $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with almost maximal mixing $(\Delta m^2_{\text{atm}}, \sin^2 2\theta_{\text{atm}}) \simeq (10^{-2.5}\text{eV}^2, 1.0)$. Again the Superkamiokande group has announced [10] that the possibility of pure sterile neutrino oscillations $\nu_\mu \leftrightarrow \nu_s$ is excluded at 99%CL. On the other hand, combining the final result of LSND and the negative results by E776 [14] $(\nu_\mu \to \nu_e)$, Karmen2 [15] $(\nu_\mu \to \nu_e)$ and Bugey [16] $(\bar{\nu}_e \to \bar{\nu}_e)$, the oscillation parameter satisfies $0.1\ \text{eV}^2 \lesssim \Delta m^2 \lesssim 8\ \text{eV}^2$ and $8 \times 10^{-4} \lesssim \sin^2 2\theta \lesssim 0.04$ at 99%CL. In this talk I will review the present status of four neutrino scenarios in the light of the recent Superkamiokande data of the solar and atmospheric neutrinos.

2 Mass patterns

In the case of four neutrino schemes there are two distinct types of mass patterns. One is the so-called (2+2)-scheme (Fig. 1(a)) and the other is the

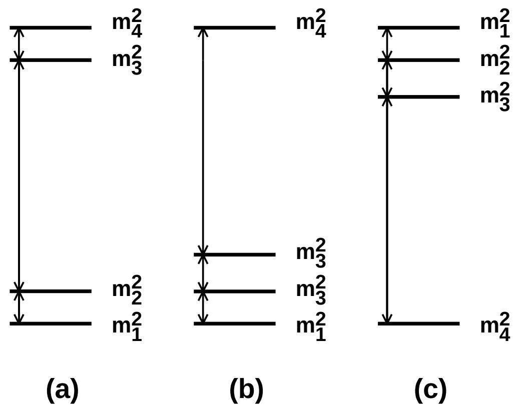

Figure 1: Mass patterns of four neutrino schemes. (a) corresponds to (2+2)-scheme, where either ($|\Delta m^2_{21}| = \Delta m^2_{\odot}$, $|\Delta m^2_{43}| = \Delta m^2_{\rm atm}$) or ($|\Delta m^2_{43}| = \Delta m^2_{\odot}$, $|\Delta m^2_{21}| = \Delta m^2_{\rm atm}$). (b) and (c) are (3+1)-scheme, where $|\Delta m^2_{41}| = \Delta m^2_{\rm LSND}$ and either ($|\Delta m^2_{21}| = \Delta m^2_{\odot}$, $|\Delta m^2_{32}| = \Delta m^2_{\rm atm}$) or ($|\Delta m^2_{32}| = \Delta m^2_{\odot}$, $|\Delta m^2_{21}| = \Delta m^2_{\rm atm}$) is satisfied.

(3+1)-scheme (Fig. 1(b) or (c)). Depending on the type of the two schemes, phenomenology is different.

2.1 (3+1)-scheme

It has been shown in Refs. [18,19] using older data of LSND [12] that the (3+1)-scheme is inconsistent with the Bugey reactor data[16] and the CDHSW disappearance experiment[17] of ν_μ. Let me briefly give this argument in Refs. [18,19]. Without loss of generality I assume that one distinct mass eigenstate is ν_4 (See Fig. 1(b) or (c)) and the largest mass squared difference is $\Delta m^2_{43} \equiv \Delta m^2_{\rm LSND}$.

In the case of (3+1)-scheme the constraints from the Bugey and CDHSW data are given by

$$\begin{aligned}1 - P(\bar{\nu}_e \to \bar{\nu}_e) &= 4|U_{e4}|^2(1-|U_{e4}|^2)\Delta_{43} \le \sin^2 2\theta_{\rm Bugey}(\Delta m^2_{43})\Delta_{43},\\ 1 - P(\nu_\mu \to \nu_\mu) &= 4|U_{\mu 4}|^2(1-|U_{\mu 4}|^2)\Delta_{43} \le \sin^2 2\theta_{\rm CDHSW}(\Delta m^2_{43})\Delta_{43},\end{aligned}$$

respectively, where $\Delta_{43} \equiv \sin^2\left(\Delta m^2_{43}L/4E\right)$, $\sin^2 2\theta_{\rm Bugey}$ and $\sin^2 2\theta_{\rm CDHSW}$ stand for the values of the boundary of the excluded region in the two flavor analysis as functions of Δm^2 (See Fig. 2). To explain the solar neutrino deficit and the zenith angle dependence of the atmospheric neutrino data it is

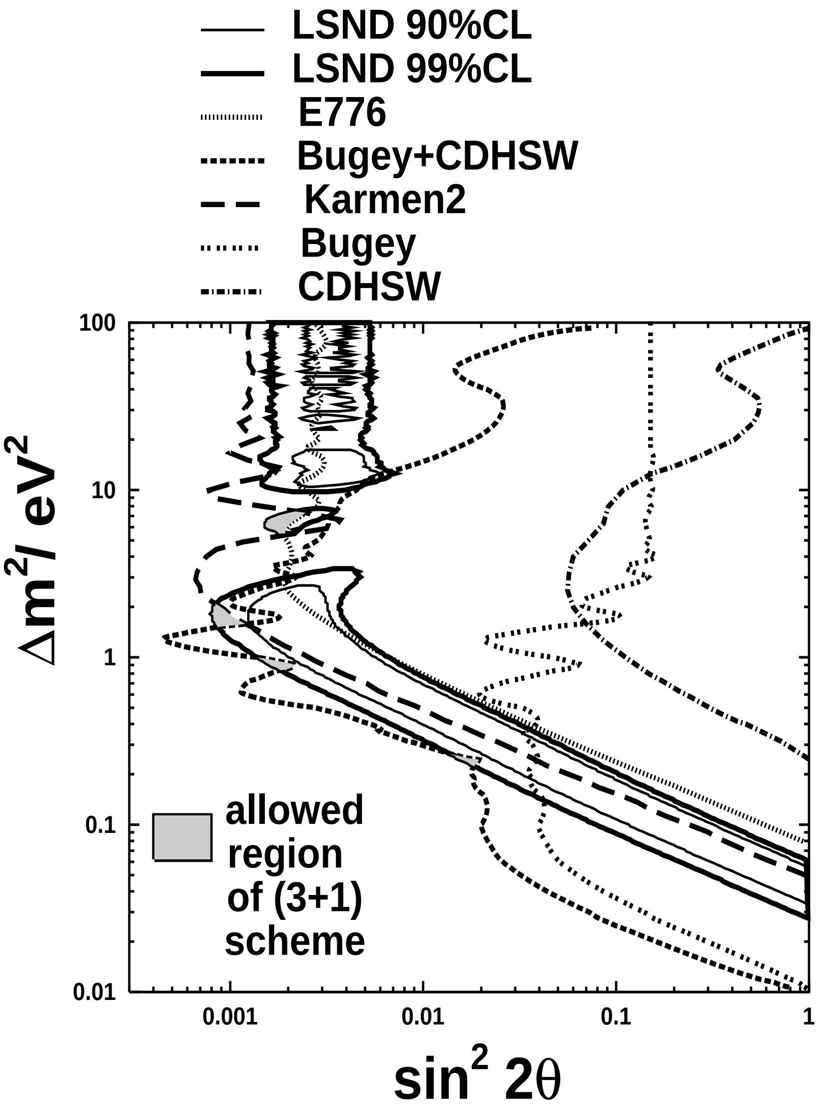

Figure 2: The allowed region of LSND from the final result (the inside of the thick and thin solid lines corresponds to the allowed region at 99%CL and 90%CL, respectively) and the excluded regions of E776, Karmen2, Bugey, CDHSW (the right hand side of each line). The right hand side of the line (Bugey+CDHSW) stands for the excluded region in the case of the (3+1)-scheme. Only the four isolated shadowed areas at $\Delta m^2_{\rm LSND}$ $\simeq$0.3, 0.9, 1.7, 6.0 eV^2 are consistent with the LSND allowed region at 99%CL in the (3+1)-scheme.

necessary to have $|U_{e4}|^2 < 1/2$ and $|U_{\mu 4}|^2 < 1/2$ and therefore I get

$$
\begin{aligned}
|U_{e4}|^2 &\le \frac{1}{2}\left[1-\sqrt{1-\sin^2 2\theta_{\rm Bugey}(\Delta m^2_{43})}\right]\\
|U_{\mu 4}|^2 &\le \frac{1}{2}\left[1-\sqrt{1-\sin^2 2\theta_{\rm CDHSW}(\Delta m^2_{43})}\right].
\end{aligned}
\tag{1}
$$

On the other hand, the appearance probability $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ of LSND in our scenario is given by

$$
P(\bar{\nu}_\mu \to \bar{\nu}_e) = 4|U_{e4}|^2|U_{\mu 4}|^2\Delta_{43} \equiv \sin^2 2\theta_{\rm LSND}(\Delta m^2_{43})\Delta_{43},
\tag{2}
$$

where $\sin^2 2\theta_{\rm LSND}(\Delta m^2_{43})$ stands for the value of $\sin^2 2\theta$ in the LSND allowed region in the two flavor framework. From (1) and (2) I obtain

$$
\begin{aligned}
\sin^2 2\theta_{\rm LSND}(\Delta m^2_{43}) \le &\left[1-\sqrt{1-\sin^2 2\theta_{\rm Bugey}(\Delta m^2_{43})}\right]\\
&\times\left[1-\sqrt{1-\sin^2 2\theta_{\rm CDHSW}(\Delta m^2_{43})}\right].
\end{aligned}
\tag{3}
$$

The value of the right hand side of (3) is plotted in Fig. 2 together with the allowed region of LSND [13]. At 90%CL the allowed region of LSND does not satisfy the condition (3) for the (3+1)-scheme, and actually it used to be the case with older data of LSND [12] even at 99%CL [18,19]. However, in the final result the allowed region has shifted to the lower value of $\sin^2 2\theta$ and it was shown [20] that there are four isolated regions $\Delta m^2_{\rm LSND} \simeq$0.3, 0.9, 1.7, 6.0 eV^2 which satisfy the condition (3).

2.2 (2+2)-scheme

In the case of the (2+2)-scheme, assuming the mass pattern in Fig. 1 (a) with $\Delta m^2_{21} = \Delta m^2_\odot$, $\Delta m^2_{32} = \Delta m^2_{\rm atm}$, $\Delta m^2_{43} = \Delta m^2_{\rm LSND}$, the constraints from the LSND, Bugey and CDHSW data are given by

$$
\begin{aligned}
P(\bar{\nu}_\mu \to \bar{\nu}_e) &= 4|U_{e3}U^*_{\mu 3}+U_{e4}U^*_{\mu 4}|^2\Delta_{32} \equiv \sin^2 2\theta_{\rm LSND}(\Delta m^2_{32})\Delta_{32},\\
1-P(\bar{\nu}_e \to \bar{\nu}_e) &= 4(|U_{e3}|^2+|U_{e4}|^2)(1-|U_{e3}|^2-|U_{e4}|^2)\Delta_{32}\\
&\le \sin^2 2\theta_{\rm Bugey}(\Delta m^2_{32})\Delta_{32},\\
1-P(\nu_\mu \to \nu_\mu) &= 4(|U_{\mu 3}|^2+|U_{\mu 4}|^2)(1-|U_{\mu 3}|^2-|U_{\mu 4}|^2)\Delta_{32}\\
&\le \sin^2 2\theta_{\rm CDHSW}(\Delta m^2_{32})\Delta_{32},
\end{aligned}
\tag{4}
$$

where $\Delta_{32} \equiv \sin^2\left(\Delta m^2_{32}L/4E\right)$.

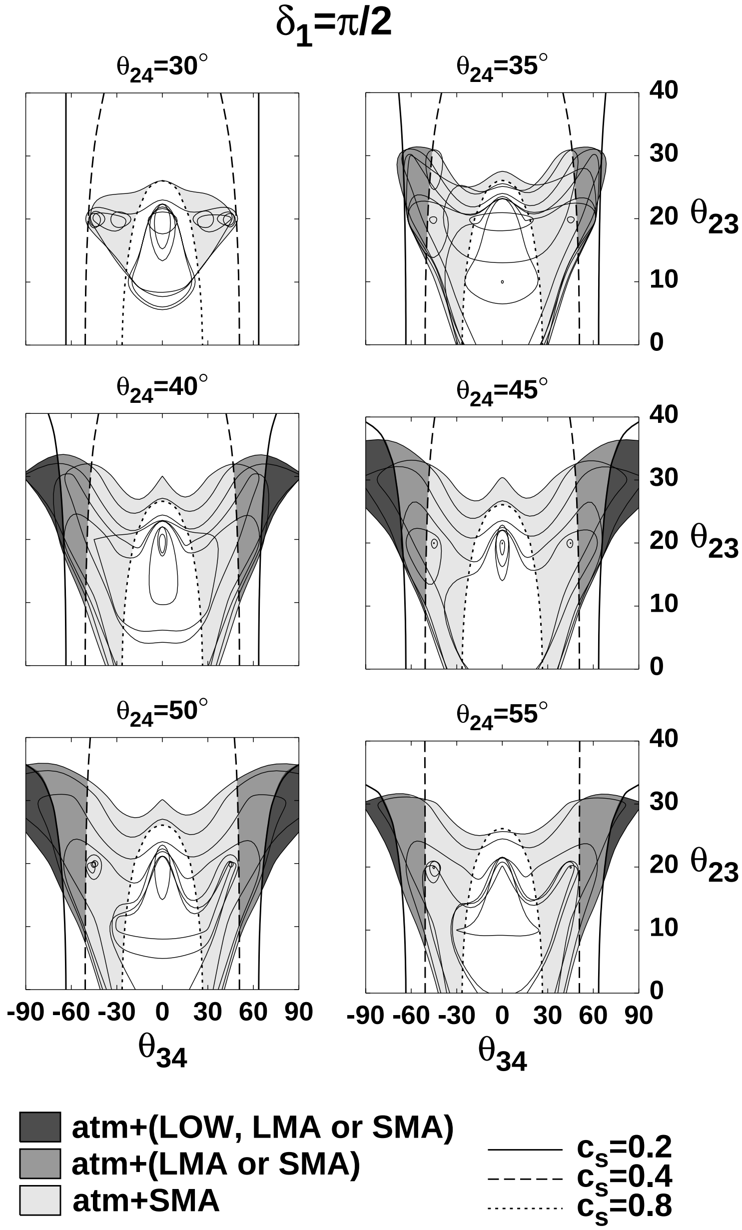

Figure 3: The allowed region of the (2+2)-scheme with $\delta_1 = \pi/2$ at 90 %CL constrained by the Superkamiokande atmospheric neutrino data. The shadowed regions stand for the allowed regions from the combined analysis of the solar and atmospheric neutrino data.

It has been shown [18,19] that these conditions are consistent with all the constraints from the accelerator, reactor data as well as solar and atmospheric neutrino observations. As I will show, to account for both the solar neutrino deficit and the atmospheric neutrino anomaly, it is necessary to have

$$4(|U_{\mu 3}|^2 + |U_{\mu 4}|^2)(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2) \sim \mathcal{O}(1), \tag{5}$$

so I take Δm^2_{32} as small as possible, i.e., $\Delta m^2_{32} = 0.3$ eV^2 so that (5) be consistent with the CDHSW constraint (4).

3 (2+2)-scheme

3.1 Analysis of the solar neutrino data

The solar neutrino data were analyzed in the framework of the (2+2)-scheme by Ref. [22] on the assumption that $U_{e3} = U_{e4} = 0$, which is justified from the Bugey constraint $|U_{e3}|^2 + |U_{e4}|^2 \ll 1$, and $\Delta m^2_{\rm atm}, \Delta m^2_{\rm LSND} \to \infty$ which is also justified since $|\Delta m^2_{\rm atm}/2E|$, $|\Delta m^2_{\rm LSND}/2E| \gg \sqrt{2} G_F N_e$ for the solar neutrino problem, where G_F and N_e stand for the Fermi constant and the electron density in the Sun. The conclusion of Ref. [22] is that the SMA MSW solution exists for $0 \le c_s \lesssim 0.8$, while the LMA MSW and LOW solutions survive only for $0 \le c_s \lesssim 0.4$ and $0 \le c_s \lesssim 0.2$, respectively, where $c_s \equiv |U_{s1}|^2 + |U_{s2}|^2$.

3.2 Analysis of the atmospheric neutrino data

The atmospheric neutrino data were analyzed by Refs. [23,24] with the (2+2)-scheme. Refs. [23,24] assumed $U_{e3} = U_{e4} = 0$ as in Ref. [22], and $\Delta m^2_\odot = 0$ was also assumed. Ref. [23] assumed $\Delta m^2_{\rm LSND} = 0.3$ eV^2 so that the result with large $|U_{\mu 3}|^2 + |U_{\mu 4}|^2$ do not contradict with the CDHSW constraint (4). Ref. [24] did not take into account the contribution from $\Delta m^2_{\rm LSND}$ to the oscillation probability and their result is a subset of Ref. [23].

Here I adopt the notation in Ref. [18] for the 4×4 MNS matrix:

$$U_{MNS} \equiv R_{34}(\frac{\pi}{2} - \theta_{34}) R_{24}(\theta_{24}) R_{23}(\frac{\pi}{2}) U_{23}(\theta_{23}, \delta_1) U_{14}(\theta_{14}, \delta_3) U_{13}(\theta_{13}, \delta_2) R_{12}(\theta_{12}) \tag{6}$$

where $U_{23}(\theta_{23}, \delta_1) \equiv e^{2i\delta_1\lambda_3} R_{23}(-\theta_{23}) e^{-2i\delta_1\lambda_3}$, $U_{14}(\theta_{14}, \delta_3) \equiv e^{\sqrt{6}i\delta_3\lambda_{15}/2} R_{14}(\theta_{14})$ $e^{-\sqrt{6}i\delta_3\lambda_{15}/2}$, $U_{13}(\theta_{13}, \delta_2) \equiv e^{2i\delta_2\lambda_8/\sqrt{3}} R_{13}(\theta_{13}) e^{-2i\delta_2\lambda_8/\sqrt{3}}$, $R_{jk}(\theta) \equiv \exp(iT_{jk}\theta)$, $(T_{jk})_{\ell m} = i\,(\delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell})$, $2\lambda_3 \equiv {\rm diag}(1, -1, 0, 0)$, $2\sqrt{3}\lambda_8 \equiv {\rm diag}(1, 1, -2, 0)$, $2\sqrt{6}\lambda_{15} \equiv {\rm diag}(1, 1, 1, -3)$ are 4×4 matrices (λ_j are elements of the $su(4)$ generators). With the assumptions $\Delta m^2_{21} = 0$, $|U_{e4}|^2 = s^2_{14} = 0$, $|U_{e3}|^2 = c^2_{14} s^2_{13} =$

0, θ_{12}, θ_{13} and θ_{14} disappear from U and ν_e decouples from other three neutrinos. Thus the problem is reduced to the three flavor neutrino analysis among ν_μ, ν_τ, ν_s and the reduced MNS matrix is

$$\widetilde{U} \equiv \begin{pmatrix} U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s2} & U_{s3} & U_{s4} \end{pmatrix} = e^{i(\frac{\pi}{2}-\theta_{34})\lambda_7} D^{-1} e^{i\theta_{24}\lambda_5} D \; e^{i(\theta_{23}-\frac{\pi}{2})\lambda_2},$$

with $D \equiv \mathrm{diag}\left(e^{i\delta_1/2}, 1, e^{-i\delta_1/2}\right)$ (λ_j are the 3×3 Gell-Mann matrices) is the reduced 3×3 MNS matrix. This MNS matrix $\widetilde{U}$ is obtained by substitution $\theta_{12} \to \theta_{23} - \pi/2$, $\theta_{13} \to \theta_{24}$, $\theta_{12} \to \pi/2 - \theta_{34}$, $\delta \to \delta_1$ in the standard parametrization in Ref. [27]. It turns out that θ_{34} corresponds to the mixing of $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$, while θ_{23} is the mixing of the contribution of $\sin^2\left(\Delta m^2_{\mathrm{atm}} L/4E\right)$ and $\sin^2\left(\Delta m^2_{\mathrm{LSND}} L/4E\right)$ in the oscillation probability. The allowed region at 90%CL of the atmospheric neutrino data is given by the area bounded by thin solid lines in Fig. 3 for $\delta_1 = \pi/2$. The allowed regions for $\delta_1 = 0, \pi/4$ are given in Ref. [23].

3.3 Combined analysis of the solar and atmospheric neutrino data

In Fig. 3, the lines given by $c_s \equiv |U_{s1}|^2 + |U_{s2}|^2 = |c_{23}c_{34} + s_{23}s_{34}s_{24}e^{i\delta_1}|^2 = 0.2, 0.4, 0.8$ are depicted together with the allowed region of the atmospheric neutrino data. By combining the analyses of Ref. [22] and Ref. [23], I obtain the region which satisfies the constraints of the solar and atmospheric neutrino data. The darkest, medium and lightest shadowed areas stand for $\nu_{\mathrm{atm}} + \nu_\odot$(SMA, LMA or LOW), $\nu_{\mathrm{atm}} + \nu_\odot$(SMA or LMA), $\nu_{\mathrm{atm}} + \nu_\odot$(SMA), respectively. Although this result is not quantitative, it gives us a sense on how likely the (2+2)-scheme is allowed by combining the solar and atmospheric neutrino data. Let me emphasize that non-zero contribution of $\sin^2\left(\Delta m^2_{\mathrm{LSND}} L/4E\right)$ (i.e., the case of $\theta_{23} > 0$) to the oscillation probability is important particularly for the LMA and LOW solar solutions. The region of $\theta_{23} > 0$ has not been analyzed by Ref. [24]. Let me also stress that both the solar neutrinos and the atmospheric neutrinos are accounted for by hybrid of active and sterile oscillations in the (2+2)-scheme.

4 (3+1)-scheme

After the work of Barger et al. [20], people [25,26] have investigated various consequences of the (3+1)-scheme. Here let me make two comments on the (3+1)-scheme.

4.1 Atmospheric neutrinos

As in the case of the (2+2)-scheme, I assume $U_{e3} = U_{e4} = \Delta m^2_\odot = 0$ for simplicity. Then ν_e once again decouples from ν_e, ν_μ, ν_τ and the probability in vacuum for the atmospheric neutrino scale is given by

$$\begin{aligned}
P(\nu_\mu \to \nu_\mu) &= 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2-|U_{\mu 4}|^2)\Delta_{32} + 2|U_{\mu 4}|^2(1-|U_{\mu 4}|^2),\\
P(\nu_\mu \to \nu_\tau) &= 4\Re\left[U_{\mu 3}U^*_{\tau 3}(U^*_{\mu 3}U_{\tau 3}+U^*_{\mu 4}U_{\tau 4})\right]\Delta_{32} + 2|U_{\mu 4}|^2|U_{\tau 4}|^2,\\
P(\nu_\mu \to \nu_s) &= 4\Re\left[U_{\mu 3}U^*_{s 3}(U^*_{\mu 3}U_{s 3}+U^*_{\mu 4}U_{s 4})\right]\Delta_{32} + 2|U_{\mu 4}|^2|U_{s 4}|^2,
\end{aligned} \tag{7}$$

where I have taken $\Delta m^2_{32} \equiv \Delta m^2_{\rm atm}$, $\Delta m^2_{43} \equiv \Delta m^2_{\rm LSND}$ and I have averaged over rapid oscillations: $\sin^2\left(\Delta m^2_{\rm LSND}L/4E\right) \to 1/2$. Since the (3+1)-scheme is allowed only for four discrete values of $\Delta m^2_{\rm LSND}$, let me discuss $\Delta m^2_{\rm LSND}$=0.3 eV^2 ($|U_{\mu 4}|^2 \gtrsim 0.34$) and $\Delta m^2_{\rm LSND}$=0.9 eV^2 ($|U_{\mu 4}|^2 \simeq 0.03$), 1.7 eV^2 ($|U_{\mu 4}|^2 \simeq 0.01$), 6.0 eV^2 ($|U_{\mu 4}|^2 \simeq 0.02$), separately. For simplicity I assume δ_1=0 since the existence of the CP phase δ_1 does not change the situation very much.

4.1.1 $\Delta m^2_{\rm LSND}$=0.3 eV^2

Since we know from the Superkamiokande atmospheric neutrino data that the coefficient of $\sin^2\left(\Delta m^2_{\rm atm}L/4E\right)$ in $P(\nu_\mu \to \nu_\mu)$ has to be large to have a good fit, I optimize $4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2-|U_{\mu 4}|^2)$ with respect to θ_{23} for $|U_{\mu 4}|^2 = 0.34$. When $U_{e4}=0$ I have $|U_{\mu 3}|^2 = c^2_{23}c^2_{24}$, $|U_{\mu 4}|^2 = s^2_{24}$, $|U_{\tau 4}|^2 = c^2_{24}c^2_{34}$, $|U_{s4}|^2 = c^2_{24}s^2_{34}$ in the notation of Ref. [18], and it is easy to see

$$4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2-|U_{\mu 4}|^2) = c^4_{24}\sin^2 2\theta_{23} \le c^4_{24} = 0.44,$$

where equality holds when $\theta_{23}=\pi/4$. This is the value of θ_{23} for which the fit of the (3+1)-scheme to the atmospheric neutrino data is supposed to be the best for $\Delta m^2_{\rm LSND}$=0.3 eV^2. When $\theta_{23}=\pi/4$ the probability in vacuum becomes

$$\begin{aligned}
P(\nu_\mu \to \nu_\tau) &= \left(c^2_{24}s^2_{34} - \frac{1}{4}\sin^2 2\theta_{24}c^2_{34}\right)\Delta_{32} + \frac{1}{2}c^2_{34}\sin^2 2\theta_{24}\\
P(\nu_\mu \to \nu_s) &= \left(c^2_{24}c^2_{34} - \frac{1}{4}\sin^2 2\theta_{24}s^2_{34}\right)\Delta_{32} + \frac{1}{2}s^2_{34}\sin^2 2\theta_{24}.
\end{aligned} \tag{8}$$

From (8) θ_{34} turns out to be the mixing of $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ as in the (2+2)-scheme.

I found from the explicit numerical calculation [28] that the fit of the (3+1)-scheme with $\Delta m^2_{\rm LSND}$=0.3 eV^2, $|U_{\mu 4}|^2 = 0.34$, $\theta_{23} = \pi/4$ to the atmospheric neutrino data is very bad for any value of θ_{34} and the region of $\Delta m^2_{\rm LSND}$=0.3 eV^2 is excluded at 6.9σCL.

4.1.2 $\Delta m^2_{\rm LSND}$ =0.9, 1.7, 6.0 eV^2

In this case $|U_{\mu 4}|^2 \lesssim 0.03$ and I can put $U_{\mu 4} = 0$ as a good approximation. Then the constant part in the oscillation probability disappears and this case is reduced to the analysis in the (2+2)-scheme with $\theta_{23} = 0$. The allowed region at 90%CL is given roughly by $-\pi/4 \lesssim \theta_{34} \lesssim \pi/4$, $0.8 \lesssim \sin^2 2\theta_{24} \leq 1$, where θ_{34} and θ_{24} stand for the mixing of $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ and the mixing of atmospheric neutrino oscillations, respectively.

4.2 Oscillations of high energy neutrinos in matter

When $|U_{e4}|^2$, $|U_{\mu 4}|^2$ and $|U_{\tau 4}|^2$ are all small, it is naively difficult to distinguish the (3+1)-scheme from the ordinary three flavor scenario. However, because of the existence of the small mixing angles in U_{e4}, $U_{\mu 4}$ and the large mass squared difference $\Delta m^2_{\rm LSND}$ the oscillation probability in matter can have enhancement which never happens in the three flavor case. By taking $\theta_{12} = \pi/4$, $\theta_{13} = 0$, $\theta_{23} = \pi/4$, $\theta_{14} = \epsilon$ ($|\epsilon| \ll 1$), $\theta_{24} = \delta$ ($|\delta| \ll 1$), $\theta_{34} = \pi/2$ in (6) I get

$$U \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \epsilon \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \delta \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{\epsilon}{\sqrt{2}} - \frac{\delta}{2} & -\frac{\epsilon}{\sqrt{2}} + \frac{\delta}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

which is the same as the MNS matrix in Ref. [20] up to the phase of each factor. The probability $P(\nu_\mu \to \nu_\mu)$ turns out to receive significant deviation from the vacuum one due to the matter effect for $E_\nu \sim \mathcal{O}(1)$ TeV, and the behaviors of $1 - P(\nu_\mu \to \nu_\mu)$ are shown in Fig. 4, where three cases of $|\Delta m^2_{\rm LSND}|$=0.9 eV^2 ($8.8^\circ \leq \epsilon \leq 12.2^\circ$, $6.4^\circ \leq \delta \leq 8.9^\circ$), 1.7 eV^2 ($7.5^\circ \leq \epsilon \leq 10.2^\circ$, $5.6^\circ \leq \delta \leq 7.7^\circ$), 6.0 eV^2 ($7.5^\circ \leq \epsilon \leq 7.7^\circ$, $10.0^\circ \leq \delta \leq 10.2^\circ$) are considered.[a] The appearance channel which is enhanced is dominantly $\nu_\mu \to \nu_s$, so it may be difficult to detect signs of this enhancement from observations of high energy

[a] The eigenvalues of the mass matrix in this case turn out to be roots of a cubic equation and analytic treatment of the oscillation probability is difficult, unlike the cases of three flavors [29] or four flavors [30,25], where one mass scale is dominant and the eigenvalues are roots of a quadratic equation.

cosmic neutrinos of energy $E_\nu \sim \mathcal{O}(1)$ TeV, although this enhancement may be observed through neutral current interactions in the future.[b]

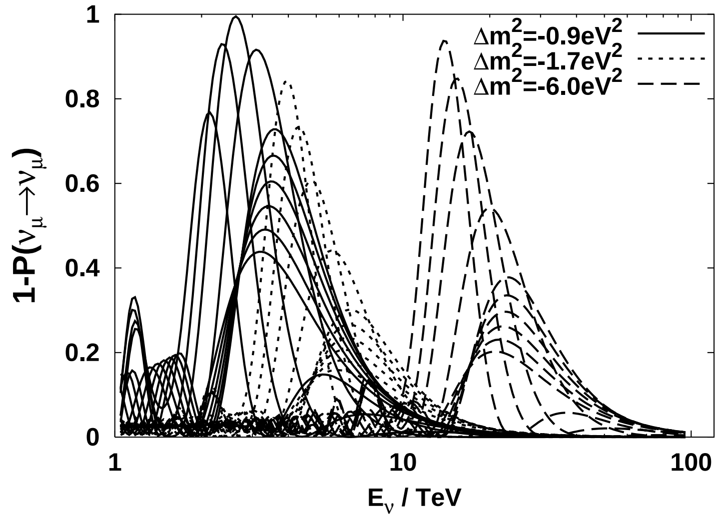

Figure 4: Enhancement of neutrino oscillations due to matter effect in the case of the (3+1)-scheme, where $\theta_{23} = 45°$ is taken for all the cases. For each value of Δm^2= -0.9 eV^2 ($\epsilon = 10.0°$, $\delta = 7.5°$), -1.7 eV^2 ($\epsilon = 5.6°$, $\delta = 7.5°$), -6.0 eV^2 ($\epsilon = 6.4°$, $\delta = 8.8°$), ten curves correspond to $\cos\Theta = -1.0, -0.9, \cdots, -0.1$ from the left to the right, where the zenith angle Θ is related to the baseline L by $L = -2R\cos\Theta$ with R=6378km. Most of the channel is $\nu_\mu \to \nu_s$.

5 Big Bang Nucleosynthesis

It has been shown [32] in the two flavor framework that if sterile neutrino have oscillations with active ones and if $\Delta m^2 \sin^4 2\theta \gtrsim 3 \times 10^{-3}$ eV^2 is satisfied then sterile neutrinos would have been in thermal equilibrium and the number N_ν of light neutrinos in Big Bang Nucleosynthesis (BBN) would be 4. This argument was generalized to the four neutrino case [18,21] and by imposing all the constraints from accelerators, reactors, solar neutrinos and atmospheric neutrinos as well as the BBN constraint $N_\nu < 4.0$ it was concluded that the only consistent four neutrino scenario is the (2+2)-scheme with the MNS mixing

[b]Similar enhancement has been discussed in a different context by Ref. [31]. I thank Athar Husain for bringing my attention to Ref. [31].

matrix

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \simeq \begin{pmatrix} c_\odot & s_\odot & \epsilon & \epsilon \\ \epsilon & \epsilon & 1/\sqrt{2} & 1/\sqrt{2} \\ \epsilon & \epsilon & -1/\sqrt{2} & 1/\sqrt{2} \\ -s_\odot & c_\odot & \epsilon & \epsilon \end{pmatrix}, \tag{9}$$

where $c_\odot \equiv \cos\theta_\odot$, $s_\odot \equiv \sin\theta_\odot$ and $\theta_\odot$ stands for the mixing angle of the SMA MSW solar solution. In this case $c_s \equiv |U_{s1}|^2 + |U_{s2}|^2 \simeq 1$ and the solar neutrino deficit would be accounted for by sterile neutrino oscillations $\nu_e \leftrightarrow \nu_s$ with the SMA MSW solution while the atmospheric neutrino anomaly would be by $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. This scenario is obviously inconsistent with the recent solar neutrino data by the Superkamiokande group, and the argument which has lead to (9) has to be given up.

Fortunately the upper bound on N_ν has become less stringent now and $N_\nu = 4.0$ seems to be allowed. Furthermore, it has been shown recently [33] that the combined analysis of BBN and the recent data by BOOMERanG [34] and MAXIMA-1 [35] of the Cosmic Microwave Background prefers higher value of N_ν: $4 \leq N_\nu \leq 13$. Therefore all the four neutrino schemes of type (2+2) and (3+1) seemed to be consistent with the BBN constraint.

On the other hand, it has been pointed out [36] in the two flavor framework that for a certain range of the oscillation parameters neutrino oscillations themselves create asymmetry between ν and $\bar{\nu}$ and this asymmetry prevents ν_s from oscillating into active neutrinos. Although this analysis has not been generalized to the four neutrino cases, even if the upper bound of N_ν becomes less than 4.0 in the future, it might be still possible to have four neutrino schemes which are consistent with the BBN constraint as well as the solar and atmospheric neutrino data due to possible asymmetry in ν and $\bar{\nu}$.

6 Conclusions

In this talk I have shown that there are still four neutrino scenarios ((2+2)- as well as (3+1)- schemes) which are consistent with all the experiments and the observations, despite the recent claims by the Superkamiokande group that pure sterile oscillations $\nu_e \leftrightarrow \nu_s$ in the solar neutrinos and pure sterile oscillations $\nu_\mu \leftrightarrow \nu_s$ in the atmospheric neutrinos are disfavored. In particular, the reason that the (2+2)-scheme is consistent with the recent Superkamiokande data is because both solar and atmospheric neutrinos have hybrid oscillations of active and sterile oscillations.

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